Understanding Compressibility of a flow

Comprendiendo la comprensibilidad de un flujo

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Abstract

The coverage of the physical phenomena experienced in compressible flow is somehow difficult to understand. However, here we provide a brief explanation of the assumptions used in the analysis of compressible flows. A strong foundation for more advanced and focused study and understanding of what causes compressible flows to differ from incompressible flows and how they can be analyzed is vital to resolve realistic problems in aerodynamics. Compressibility issues arise when fluids are moving at velocities higher than the speed of sound which is the equivalent of Mach number 0.3. It is vital to understand the behavior of the fluid beyond this number because compressibility factors affect the fluid and it would be necessary to account for such factors in order to get results closer to the real solution.

Keywords: compressible, flow, incompressible, aerodynamics, Mach number, speed, sound

Resumen

La cobertura de los fenómenos físicos experimentados en el flujo compresible es algo difícil de entender. Sin embargo, aquí damos una explicación breve acerca de las hipótesis empleadas en el análisis de flujo compresible. Una base sólida para un estudio más avanzado y enfocado, y la comprensión de las causas de flujo compresible para diferenciar de los flujos incompresibles y la forma en que se pueden analizar es vital para resolver problemas reales en aerodinámica. Los asuntos de comprensibilidad aparecen cuando los fluidos se están moviendo con una velocidad mayor que la velocidad del sonido la cual es equivalente al valor del número Mach 0.3. Es importante entender el comportamiento del fluido por encima de este valor de número Mach porque los factores de comprensibilidad afectan al fluido y se necesitaría tomar en cuenta tales factores para conseguir resultados cercanos a la solución real.

Palabras clave: compresible, flujo, incompresible, aerodinámica, número Mach, velocidad, sonido

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Introduction

High speed flows are frequently encountered in engineering applications. Flow features such as shock waves in nozzles or supersonic wind tunnels, expansion waves, and oblique or bow shock waves in front of rapidly moving objects are all examples of intriguing phenomena that occur due to high speeds and compressibility of fluids. Compressible fluid flow theory is intended to cover compressible behavior of fluids. Here, we provide a brief introduction to compressible flows, including fundamentals of isentropic compressibility and isothermal compressibility. This phenomenon takes place in certain regions of speed of the flow such as high subsonic, transonic, sonic, supersonic and hypersonic velocities.

However, normal shocks, oblique and expansion shocks may appear in these phenomena contributing to the complexity of the solution. Properties of the flow may change dramatically right after the shocks affecting the whole solution of the fluid flow in this area and the surrounding

Discussion

Compressible flow is routinely defined as variable density flow; this is in contrast to incompressible flow, where the density is assumed to be constant throughout. Obviously, in real life every flow of every fluid is compressible to some greater or lesser extent; hence, a truly constant density (incompressible) flow is a myth. However, as previously mentioned, for almost all liquid flows as well as for the flows of some gases under certain conditions, the density changes are so small that the assumption of constant density can be made with reasonable accuracy. In such cases, Bernoulli's equation 1, defined by Anderson, Jr. (1989) can be applied with confidence.

$$p + \frac{1}{2}\rho V^2 = const \qquad (1)$$

However, the simple definition of compressible flow as one in which the density is variable requires more elaboration. Consider a small element of fluid of volume "v" as shown in Figure 1. The pressure exerted on the sides of the element by the neighboring fluid is p.
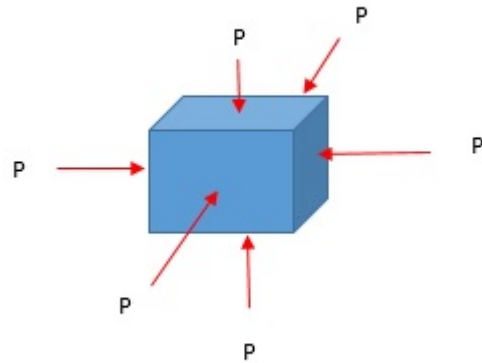

Figure 1. Small element of fluid of volume v

Assume the pressure is now increased by an infinitesimal amount dp. The volume of the element will be correspondingly compressed by the amount dv. Since the volume is reduced, dv is a negative quantity. The compressibility of the fluid, Ʈ, is defined by Anderson, Jr. (2003) as

$$\tau = -\frac{1}{v}\frac{dv}{dp} \qquad (2)$$

Physically, the compressibility is the fractional change in volume of the fluid element per unit change in pressure. However, equation 2 is not sufficiently precise. We know from experience that when a gas is compressed, its temperature tends to increase, depending on the amount of heat transferred into or out of the gas through the boundaries of the system. Therefore, if the temperature of the fluid element is held constant (due to some heat transfer mechanism), then the isothermal compressibility is defined by Anderson, Jr. (2003) as

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T \qquad (3)$$

On the other hand, if no heat is added to or taken away from the fluid element (if the compression is adiabatic), and if no other dissipative transport mechanisms such as viscosity and diffusion are important (if the compression is reversible), then the compression of the fluid element takes place isentropically as described by Shapiro (1953), and the isentropic compressibility is defined by Anderson, Jr. (2003) as

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s \qquad (4)$$

where the subscript s denotes that the partial derivative is taken at constant entropy.

Compressibility is a property of the fluid. Liquids have very low values of compressibility (τ_T for water is 5×10^{-10} m²/ N at 1 atm) whereas gases have high compressibility (τ_T for air is 10^{-5} m²/ N at 1 atm). If the fluid element is assumed to have unit mass, v is the specific volume (volume per unit mass), and the density is

$$\rho = \frac{1}{v} \qquad (5)$$

In terms of density, equation 2 becomes

$$\tau = \frac{1}{\rho}\left(\frac{d\rho}{dp}\right) \qquad (6)$$

Therefore, whenever the fluid experiences a change in pressure, dp, the corresponding change in density will be dρ, where from equation 6

$$d\rho = \rho\tau dp \qquad (7)$$

To this point, we have considered just the fluid itself, with compressibility being a property of the fluid. Now assume that the fluid is in motion. According to Zucrow and Hoffman (1976), such flows are initiated and maintained by forces on the fluid, usually created by, or at least accompanied by, chan-

ges in the pressure. In particular, we shall see that high-speed flows generally involve large pressure gradients. For a given change in pressure, dp, due to the flow, equation 7 demonstrates that the resulting change in density will be small for liquids (which have low values of τ), and large for gases (which have high values of τ). Therefore, for the flow of liquids, relatively large pressure gradients can create high velocities without much change in density. Hence, such flows are usually assumed to be incompressible, where ρ is constant. On the other hand, for the flow of gases with their attendant large values of τ, moderate to strong pressure gradients lead to substantial changes in the density via equation 7. At the same time, such pressure gradients create large velocity changes in the gas. Such flows are defined as compressible flows, where ρ is a variable.

If the velocity of gases is less than 0.3 than the speed of sound (M<0.3), the associated pressure changes are small, and even though τ is large for gases, dp in equation 7 may still be small enough to dictate a small dρ. For this reason, the low-speed flow of gases can be assumed to be incompressible as shown in Figure 2. This is velocities less than 250 mi/h (112 m/s). On the other hand, for flow velocities higher than 0.3 of the speed of sound (M>0.3), the associated pressure changes, dp are relatively large, and having a large value of τ for gases, large changes in density, dρ are produced via equation 7
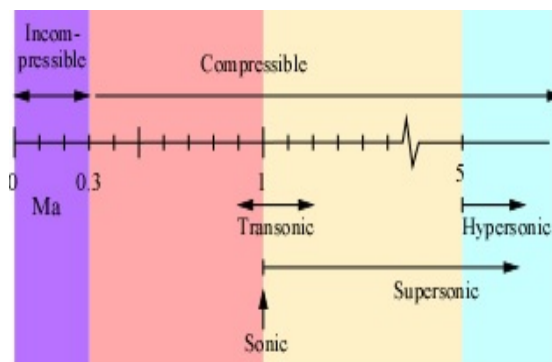

Figure 2. Compressibility Range of 0.3 M

Conclusion

In summary, a compressible flow is considered as one where the change in pressure, dp, over a characteristic length of the flow, multiplied by the compressibility via equation 7, results in a fractional change in density, $d\rho/\rho$, which is too large to be ignored. For most practical problems, if the density changes by 5% or more, the flow is considered to be compressible.

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